# Final Gathering using Adaptive Multiple Importance Sampling (sap\_0079)

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Figure 1: Cornell box with a point light source near the ceiling. Left: classic final gathering. Right: our method. The same number of samples is used for both images ( $512 \times 512$  pixels, 4 oversamples/pixel, 1024 final gather rays/pixel). The number of iterations for AMIS is 4. The rendering times for left and right are 116.5 seconds and 134.5 seconds, respectively (CPU: Xeon W5590). Our method produces less noise than the classic final gathering with small overhead.

### 1 Introduction

We propose an efficient final gathering technique using adaptive multiple importance sampling (AMIS) [Cornuet et al. 2009] for a scene containing a highly intense spot of light. AMIS is aimed at optimally recycling past simulations in an iterative importance sampling scheme. The difference to earlier adaptive importance sampling methods is that the past weighting functions are recomputed by multiple importance sampling [Veach 1997] at each iteration. In AMIS, the probability distribution function (PDF) at the *t*th iteration is parameterized by  $\hat{\theta}_t$ . The next parameter  $\hat{\theta}_{t+1}$  is determined by estimating the optimal value from past samples (described in Section 2.2). The performance of AMIS depends on a sampling strategy, i.e. the choice of a PDF. This poster suggests a suitable PDF for final gathering. Our method increases error in some case, however, it is effective in the case of a scene contains a highly intense spot of light compared to the classic method.

## 2 Sampling Final Gather Rays

### 2.1 PDF on Diffuse Surfaces

Usually, final gather rays are sampled according to the PDF of a reflectance model. If it is diffuse, the PDF of the lambertian model:  $p(\boldsymbol{\omega}) = (\boldsymbol{\omega} \cdot \boldsymbol{n}_s)/\pi$  is often used ( $\boldsymbol{\omega}$ : ray direction,  $\boldsymbol{n}_s$ : surface normal). We extend the PDF p with the parameter  $\hat{\boldsymbol{\theta}}_t = (\hat{\boldsymbol{n}}_t, \hat{\alpha}_t)$  as

$$p(\boldsymbol{\omega}; \hat{\boldsymbol{\theta}}_t) = \frac{\hat{\alpha}_t + 1}{2\pi} |\boldsymbol{\omega} \cdot \hat{\boldsymbol{n}}_t|^{\hat{\alpha}_t}.$$
 (1)

The initial value of  $\hat{n}_0 = n_s$  and  $\hat{\alpha}_0 = 1$ . Hence, Equation (1) is the equivalent to the lambertian model, but then optimized to the integrand in an iterative fashion. Therefore, our method is expected to generate less variance than the lambertian model.

### 2.2 Updating the PDF

Maximum likelihood estimation (MLE) is an appropriate method for determination of the next parameter  $\hat{\theta}_{t+1}$ . Let  $f(\omega)$  be the integrand, and  $w_j(\omega)$  the weighting function, and  $N_j$  the number of samples,  $\hat{\alpha}_{t+1}$  is estimated by MLE as

$$\hat{\alpha}_{t+1} = -1 - \frac{\sum_{j=0}^{t} \sum_{i=0}^{N_j-1} s_{i,j}}{\sum_{j=0}^{t} \sum_{i=0}^{N_j-1} s_{i,j} \log |\boldsymbol{\omega}_{i,j} \cdot \hat{\boldsymbol{n}}_{t+1}|}, \quad (2)$$

$$s_{i,j} = w_j(\boldsymbol{\omega}_{i,j}) \frac{f(\boldsymbol{\omega}_{i,j})}{N_j p(\boldsymbol{\omega}_{i,j}; \hat{\boldsymbol{\theta}}_j)}.$$
(3)

However, obtaining  $\hat{n}_{t+1}$  is computationally expensive with MLE. Therefore, we assume  $\hat{n}_{t+1}$  to be the average of the sampled directions, which is given as

$$\hat{n}_{t+1} = \frac{\sum_{j=0}^{t} \sum_{i=0}^{N_j - 1} s_{i,j} \omega_{i,j}}{\|\sum_{i=0}^{t} \sum_{i=0}^{N_j - 1} s_{i,j} \omega_{i,j}\|}.$$
(4)

The detail of this section is described in the supplemental material.

### 2.3 Glossy Surfaces

Sections 2.1 and 2.2 described the case for diffuse surfaces. We are also able to operate a PDF in the same manner even with glossy surfaces. For example, if a reflectance model uses the phong distribution function [Lafortune and Willems 1994], we can replace the ray direction with a halfway vector in Equations (1)(2)(4), and the  $\hat{\alpha}_0$  with the phong exponent of the reflectance model.

### **3** Experimental Results

Figure 1 shows the experimental results. The scene is a Cornell box illuminated by a point light source near the ceiling. A highly intense spot of light is made in the center of the ceiling. Our method reduces noise compared to the classic method with small overhead.

#### References

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