PDF of a Minimum Random Number

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1 PDF of a Minimum Random Number

The probability that a uniform random number $u \in [0, 1)$ is the minimum value among N uniform random numbers, is given by the following recursion:

$$P_{\min,N}(u) = (1-u)P_{\min,N-1}(u),$$

$$P_{\min,1}(u) = 1,$$

where $1-u$ is the probability that the other random number is larger than u. Expanding this recursion, we obtain the following probability:

$$P_{\min,N}(u) = (1-u)^{N-1}. \tag{1}$$

The probability density function (PDF) of the minimum random number is obtained by normalizing Eq. (1) as follows:

$$p_{\min,N}(u) = \frac{P_{\min,N}(u)}{\int_0^1 P_{\min,N}(u')\,\mathrm{d}u'} = N(1-u)^{N-1}.$$

2 Generation of a Minimum Random Number

The cumulative distribution function (CDF) of the PDF $p_{\min,N}(u)$ is yielded as

$$c_{\min,N}(u) = \int_0^u p_{\min,N}(u')\,\mathrm{d}u' = 1-(1-u)^N.$$

Using the inverse function of this CDF, the minimum random number is generated by using a single random number $\xi \in [0, 1)$ as follows:

$$c^{-1}_{\min,N}(\xi) = 1-(1-\xi)^{\frac{1}{N}}.$$

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